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JHEP10(2009)074

(http://iopscience.iop.org/1126-6708/2009/10/074)

Published by IOP Publishing for SISSA

Received: *September 25, 2009*
Accepted: *October 10, 2009*
Published: *October 26, 2009*

Leading-twist light cone distribution amplitudes for p-wave heavy quarkonium states

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Abstract: In this paper, a study of light-cone distribution amplitudes for p-wave heavy quarkonium states are presented. Within the light-front framework, the leading twist light-cone distribution amplitudes, and their relevant decay constants, have some simple relations. These relations can be further simplified when the non-relativistic limit and the wave function as a function of relative momentum $|\vec{\kappa}|$ are taken into consideration. In addition, the $\kappa_\perp$ integrations in the equations of LCDAs and ξ-moments can be completed analytically when the Gaussian-type wave function is considered. After fixing the parameters that appear in the wave function, the curves and the corresponding decay constants of the LCDAs are plotted and calculated for the charmonium and bottomonium states. The first three non-vanishing ξ-moments of the LCDAs are estimated and are consistent with those of other theoretical approaches.

Keywords: Phenomenological Models

ArXiv ePrint: 0906.4412

doi:10.1088/1126-6708/2009/10/074

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Contents

1 Introduction

Light-cone distribution amplitudes (LCDAs) of hadrons are key ingredients in the description of various exclusive processes of quantum Chromodynamics (QCD), and their role can be analogous to those of parton distributions in inclusive processes. In terms of the Bethe-Salpeter wave functions $\Phi(u_i, k_{i\perp})$, LCDAs $\phi(u_i)$ are defined by retaining the momentum fractions u_i and integrating out the transverse momenta $k_{i\perp}$ [1]. They provide essential information on the non-perturbative structure of the hadron for QCD treatment of exclusive reactions. Specifically, the leading twist LCDAs describe the probability amplitudes to find the hadron in a Fock state with the minimum number of constituents. In addition, the fact that B-physics exclusive processes are under investigation in BaBar and Belle experiments, also urges the detailed study of hadronic LCDAs. In literature, there have been many non-perturbative approaches to estimate LCDAs, such as the QCD sum rules [2–6], lattice calculation [7, 8], chiral quark model from the instanton vacuum [9, 10], Nabmbu-Jona-Lasinio model [11, 12], and the light-front quark model [13–15]. These studies have dealt with LCDAs of pseudoscalar [3, 8–14], vector [4, 7, 13, 14], axial vector [5, 6, 15], and tensor [6] mesons.

The present paper is devoted to the study of leading twist LCDAs of p-wave heavy quarkonium states which include the scalar (χ_{c0}, χ_{b0}), axial vector $(\chi_{c1}, \chi_{b1}, h_c, h_b)$, and tensor (χ_{c2}, χ_{b2}) mesons. The motivation of this study is as follows. Since the discoveries

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of J/ψ and Υ, occurring more than thirty years ago, a great deal of information on heavy quarkonium levels and their transitions has been accumulated [16]. The numerous transitions between heavy quarkonium states are classified as strong and radiative decays, which shed light on aspects of QCD in both the perturbative and the non-perturbative regimes (for a recent review see [17]). In particular, some experimental results regarding χ_{cJ} mesons have recently been reported [18–21]. Therefore, a thorough understanding of their properties, such as LCDAs which are the universal non-perturbative objects, will be of great benefit when analyzing the hard exclusive processes with heavy quarkonium production.

It is known that heavy quarkonium is relevant to non-relativistic treatments [22]. Although non-relativistic QCD (NRQCD) is a powerful theoretical tool for separating high-energy modes from low-energy contributions, in most cases the calculation of low-energy hadronic matrix elements has relied on model-dependent non-perturbative methods. In this study, heavy quarkonium is explored within the light-front quark model (LFQM) which is a promising analytic method for solving the non-perturbative problems of hadron physics [23] as well as offers many insights into the internal structures of bound states. The basic ingredient in LFQM is the relativistic hadron wave function which generalizes distribution amplitudes by including transverse momentum distributions, and contains all the information of a hadron from its constituents. The hadronic quantities are represented by the overlap of wave functions and can be derived in principle. The light-front wave function is manifestly a Lorentz invariant, expressed in terms of internal momentum fraction variables which are independent of the total hadron momentum. Moreover, the fully relativistic treatment of quark spins and center-of-mass motion can be carried out using the so-called Melosh rotation [24]. This treatment has been successfully applied to calculate phenomenologically many important meson decay constants and hadronic form factors [25–30]. Therefore, the main purpose of this study is the calculation of the leading twist LCDAs of p-wave heavy quarkonium states within LFQM.

The remainder of this paper is organized as follows. In section 2, the leading twist LCDAs of p-wave meson states are shown in cases of the vector and tensor currents. In section 3, the formulism of LFQM is reviewed briefly, then the leading twist LCDAs are extracted within LFQM. The ξ-moments of these LCDAs are also calculated. In section 4, numerical results and discussions are presented. Finally, the conclusions are given in section 5.

2 Leading twist LCDAs of p-wave mesons

Amplitudes of hard processes involving p-wave mesons can be described by the matrix elements of gauge-invariant nonlocal operators, which are sandwiched between the vacuum and the meson states,

$$\langle 0|\bar{q}(x)\Gamma[x,-x]q(-x)|H(P,\epsilon)\rangle, \tag{2.1}$$

where P is the meson momentum, ϵ is the polarization vector or tensor (of course, ϵ does not exist in the case of scalar meson), Γ is a generic notation for the Dirac matrix structure

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and the path-ordered gauge factor is:

$$[x,y] = \mathrm{P}\ \exp\left[ig_s\int_0^1 dt(x-y)_\mu A^\mu(tx+(1-t)y)\right]. \tag{2.2}$$

This factor is equal to unity in the light-cone gauge which is equivalent to the fixed-point gauge, $(x-y)_\mu A^\mu(x-y)=0$, as the quark-antiquark pair is at the light-like separation [31]. For simplicity, the gauge factor will not be shown below.

The asymptotic expansion of exclusive amplitudes, in powers of large momentum transfer, is governed by the expanding amplitude eq. (2.1), shown in powers of deviation from the light-cone $x^2=0$. There are two light-like vectors, p and z, which can be introduced by:

$$p^2=0, \qquad z^2=0, \tag{2.3}$$

such that $p\to P$ in the limit $M_H^2\to 0$ and $z\to x$ for $x^2=0$. From this it follows that [4]

$$\begin{aligned} z^\mu &= x^\mu - P^\mu\frac{1}{M_H^2}\left[Px-\sqrt{(Px)^2-x^2M_H^2}\right] \\ &= x^\mu - P^\mu\frac{x^2}{2Pz}+O(x^4), \\ p^\mu &= P^\mu - z^\mu\frac{M_H^2}{2Pz}, \end{aligned} \tag{2.4}$$

where $Px\equiv P\cdot x$ and $Pz=pz=\sqrt{(Px)^2-x^2M_H^2}$. In addition, if the meson is assumed that it moves in the positive $\hat{e}_3$ direction, then p^+ and z^- are the only nonzero component of p and z, respectively, in an infinite momentum frame. For the axial vector meson, the polarization vector ϵ^μ is decomposed into longitudinal and transverse projections as:

$$\epsilon_\parallel^\mu = \frac{\epsilon z}{pz}\left(p^\mu - z^\mu\frac{M_H^2}{2pz}\right), \quad \epsilon_\perp^\mu = \epsilon^\mu - \epsilon_\parallel^\mu, \tag{2.5}$$

respectively. For the tensor meson, the polarization tensor is:

$$\epsilon^{\mu\nu}(m) = \langle 11; m'm''|11;2m\rangle\epsilon^\mu(m')\epsilon^\nu(m''), \tag{2.6}$$

(m is the magnetic quantum number) and $\epsilon_{\mu\bullet}(\equiv \epsilon^{\mu\nu}z_\nu)$ can also be decomposed into longitudinal and transverse projections as:

$$\epsilon_\parallel^{\mu\bullet} = \frac{\epsilon^{\bullet\bullet}}{pz}\left(p^\mu - z^\mu\frac{M_H^2}{2pz}\right), \quad \epsilon_\perp^{\mu\bullet} = \epsilon^{\mu\bullet} - \epsilon_\parallel^{\mu\bullet}. \tag{2.7}$$

LCDAs are defined in terms of matrix element of nonlocal operator in eq. (2.1). For the scalar (S), axial vector (A), and tensor (T) mesons, the leading twist LCDAs can be defined as:

$$\langle 0|\bar{q}(z)\gamma^\mu q(-z)|S(P)\rangle = f_S\int_0^1 du\ e^{i\xi pz}\left[p^\mu\phi_S(u) + z^\mu\frac{M_S^2}{2pz}g_S(u)\right], \tag{2.8}$$

$$\begin{aligned} \langle 0|\bar{q}(z)\gamma^\mu\gamma_5 q(-z)|A(P,\epsilon_{\lambda=0})\rangle = if_AM_A\int_0^1 du\ e^{i\xi pz}\Bigg\{ & p^\mu\frac{\epsilon z}{pz}\phi_{A\parallel}(u) + \epsilon_\perp^\mu g_{A\perp}(u) \\ & - z^\mu\frac{\epsilon z}{2(pz)^2}M_A^2 g_{A3}(u)\Bigg\}, \end{aligned} \tag{2.9}$$

$$\langle 0|\bar{q}(z)\sigma^{\mu\nu}\gamma_5 q(-z)|A(P,\epsilon_{\lambda=\pm1})\rangle = f_A^\perp \int_0^1 du\; e^{i\xi pz}\Big\{(\epsilon_\perp^\mu p^\nu - \epsilon_\perp^\nu p^\mu)\phi_{A\perp}(u) + (p^\mu z^\nu - p^\nu z^\mu)\frac{M_A^2\epsilon z}{(pz)^2}h_{A\parallel}(u) + (\epsilon_\perp^\mu z^\nu - \epsilon_\perp^\nu z^\mu)\frac{M_A^2}{2pz}h_{A3}(u)\Big\}, \tag{2.10}$$

$$\langle 0|\bar{q}(z)\gamma^\mu q(-z)|T(P,\epsilon_{\lambda=0})\rangle = f_T M_T^2 \int_0^1 du\; e^{i\xi pz}\Big\{p^\mu\frac{\epsilon^{\bullet\bullet}}{(pz)^2}\phi_{T\parallel}(u) + \frac{\epsilon_\perp^{\mu\bullet}}{pz}g_{T\perp}(u) - z^\mu\frac{\epsilon^{\bullet\bullet}}{2(pz)^3}M_T^2 g_{T3}(u)\Big\}, \tag{2.11}$$

$$\langle 0|\bar{q}(z)\sigma^{\mu\nu} q(-z)|T(P,\epsilon_{\lambda=\pm1})\rangle = i f_T^\perp M_T \int_0^1 du\; e^{i\xi pz}\Big\{\frac{(\epsilon_\perp^{\mu\bullet}p^\nu - \epsilon_\perp^{\bullet\nu}p^\mu)}{pz}\phi_{T\perp}(u) + (p^\mu z^\nu - p^\nu z^\mu)\frac{M_T^2\epsilon^{\bullet\bullet}}{(pz)^3}h_{T\parallel}(u) + (\epsilon_\perp^{\mu\bullet}z^\nu - \epsilon_\perp^{\bullet\nu}z^\mu)\frac{M_T^2}{2(pz)^2}h_{T3}(u)\Big\}, \tag{2.12}$$

where u is the momentum fraction and $\xi \equiv (1-u) - u = 1 - 2u$. Here ϕ_S, $\phi_{A,T\parallel}$, and $\phi_{A,T\perp}$ are the leading twist-2 LCDAs, and the others contain contributions from higher-twist operators. Due to G-parity, ϕ_S, g_S, $\phi_{^3A_1\perp}$, $h_{^3A_1\parallel}$, $h_{^3A_13}$, $\phi_{^1A_1\parallel}$, $g_{^1A_1\perp}$, $g_{^1A_13}$, $\phi_{T\parallel}$, $g_{T\perp}$, g_{T3}, $\phi_{T\perp}$, $h_{T\parallel}$, and h_{T3} are antisymmetric (odd) under replacement $u \to 1-u$, whereas, $\phi_{^1A_1\perp}$, $h_{^1A_1\parallel}$, $h_{^1A_13}$, $\phi_{^3A_1\parallel}$, $g_{^3A_1\perp}$, and $g_{^3A_13}$ are symmetric (even) in the quarkonium states. Therefore, the leading twist LCDAs are normalized as:

$$\int_0^1 du\xi\phi^{(\mathrm{odd})}(u) = 1, \qquad \int_0^1 du\phi^{(\mathrm{even})}(u) = 1. \tag{2.13}$$

and can be expanded [2] in Gegenbauer polynomials $C_n^{3/2}(\xi)$ as

$$\phi(\xi,\mu) = \phi_{\mathrm{as}}(\xi)\left[\sum_{l=0}^{\infty} a_l(\mu)C_l^{3/2}(\xi)\right]. \tag{2.14}$$

where $\phi_{\mathrm{as}}(\xi) = 3(1-\xi^2)/4$ is the asymptotic quark distribution amplitude and $a_l(\mu)$ are the Gegenbauer moments which describe to what degree the quark distribution amplitude deviates from the asymptotic one. $C_l^{3/2}(\xi)$s have the orthogonality integrals

$$\int_{-1}^1 (1-\xi^2)C_l^{3/2}(\xi)C_m^{3/2}(\xi)d\xi = \frac{2(l+1)(l+2)}{2l+3}\,\delta_{lm}. \tag{2.15}$$

Then a_l can be obtained by using the above orthogonality integrals as

$$a_l(\mu) = \frac{2(2l+3)}{3(l+1)(l+2)}\int_{-1}^1 C_l^{3/2}(\xi)\phi(\xi,\mu)d\xi. \tag{2.16}$$

An alternative approach to parameterize quark distribution amplitude is to calculate the so-called ξ-moments

$$\langle\xi^n\rangle_\mu = \int_{-1}^1 d\xi\;\xi^n\phi(\xi,\mu). \tag{2.17}$$

To disentangle the twist-2 LCDAs from higher twist in eqs. (2.8)–(2.12), the twist-2 contribution of the relevant nonlocal operator $\bar{q}(z)\Gamma q(-z)$ must be derived. For the

$\Gamma = \gamma^\mu(\gamma_5)$ case, the leading twist-2 contribution contains contributions of the operators which are fully symmetric in Lorentz indices [32, 33]:

$$[\bar{q}(-z)\gamma^\mu(\gamma_5)q(z)]_2 = \sum_{n=0}^{\infty}\frac{1}{n!}\bar{q}(0)\left\{\frac{(z\cdot\widehat{D})^n}{n+1}\gamma^\mu + \frac{n(z\cdot\widehat{D})^{n-1}}{n+1}\widehat{D}^\mu \not{z}\right\}(\gamma_5)q(0), \quad (2.18)$$

where $\widehat{D} = \overrightarrow{D} - \overleftarrow{D}$ and $\overrightarrow{D} = \overrightarrow{\partial} - igB^a(\lambda^a/2)$. The sum can be repressed in terms of a nonlocal operator,

$$[\bar{q}(-z)\gamma^\mu(\gamma_5)q(z)]_2 = \int_0^1 dt\frac{\partial}{\partial z_\mu}\bar{q}(-tz)\ \not{z}(\gamma_5)q(tz). \quad (2.19)$$

Taking the matrix element between the vacuum and the p-wave meson state, we obtain:

$$\langle 0|[\bar{q}(-z)\gamma^\mu q(z)]_2|S(P)\rangle = f_S\int_0^1 du\phi_S(u)\left\{p^\mu e^{i\xi pz} + (P^\mu - p^\mu)\int_0^1 dte^{i\xi tpz}\right\}, \quad (2.20)$$

$$\langle 0|[\bar{q}(-z)\gamma^\mu\gamma_5 q(z)]_2|A(P,\epsilon_{\lambda=0})\rangle = if_A M_A\int_0^1 du\phi_{A\|}(u)\left\{p^\mu\frac{\epsilon z}{pz}e^{i\xi pz}\right. \quad (2.21)$$
$$\left. + \left(\epsilon^\mu - p^\mu\frac{\epsilon z}{pz}\right)\int_0^1 dte^{i\xi tpz}\right\},$$

$$\langle 0|[\bar{q}(-z)\gamma^\mu q(z)]_2|T(P,\epsilon_{\lambda=0})\rangle = f_T M_T^2\int_0^1 du\phi_{T\|}(u)\left\{p^\mu\frac{\epsilon^{\bullet\bullet}}{(pz)^2}e^{i\xi pz}\right. \quad (2.22)$$
$$\left. + 2\left(\frac{\epsilon^{\mu\bullet}}{pz} - p^\mu\frac{\epsilon^{\bullet\bullet}}{(pz)^2}\right)\int_0^1 dte^{i\xi tpz}\right\}.$$

The derivations of eqs. (2.20)–(2.22) are shown in appendix A. We can use eq. (2.18), and then expand the right-hand sides of eqs. (2.20)–(2.22), as

$$\sum_{n=0}^{\infty}\frac{1}{n!}\langle 0|\bar{q}(0)\left\{\frac{(z\cdot\widehat{D})^n}{n+1}\gamma^\mu + \frac{n(z\cdot\widehat{D})^{n-1}}{n+1}\widehat{D}^\mu \not{z}\right\}q(0)|S(P)\rangle$$
$$= f_S\sum_{n=0}^{\infty}\frac{i^n}{n!}\int_0^1 du\phi_S(u)(\xi pz)^n\left\{p^\mu + (P^\mu - p^\mu)\int_0^1 dtt^n\right\}, \quad (2.23)$$

$$\sum_{n=0}^{\infty}\frac{1}{n!}\langle 0|\bar{q}(0)\left\{\frac{(z\cdot\widehat{D})^n}{n+1}\gamma^\mu + \frac{n(z\cdot\widehat{D})^{n-1}}{n+1}\widehat{D}^\mu \not{z}\right\}q(0)|A(P,\epsilon)\rangle$$
$$= if_A M_A\sum_{n=0}^{\infty}\frac{i^n}{n!}\int_0^1 du\phi_{A\|}(u)(\xi pz)^n\left\{p^\mu\frac{\epsilon z}{pz} + \left(\epsilon^\mu - p^\mu\frac{\epsilon z}{pz}\right)\int_0^1 dtt^n\right\}, \quad (2.24)$$

$$\sum_{n=0}^{\infty}\frac{1}{n!}\langle 0|\bar{q}(0)\left\{\frac{(z\cdot\widehat{D})^n}{n+1}\gamma^\mu + \frac{n(z\cdot\widehat{D})^{n-1}}{n+1}\widehat{D}^\mu \not{z}\right\}q(0)|T(P,\epsilon)\rangle$$
$$= f_T M_T^2\sum_{n=0}^{\infty}\frac{i^n}{n!}\int_0^1 du\phi_{T\|}(u)(\xi pz)^n\left\{p^\mu\frac{\epsilon^{\bullet\bullet}}{(pz)^2} + 2\left(\frac{\epsilon^{\mu\bullet}}{pz} - p^\mu\frac{\epsilon^{\bullet\bullet}}{(pz)^2}\right)\int_0^1 dtt^n\right\}, \quad (2.25)$$

respectively. Picking $n = 0$ in eqs. (2.23) and (2.24), we obtain

$$\langle 0|\bar{q}(0)\gamma^\mu q(0)|S(P)\rangle = f_S P^\mu \int_0^1 du\phi_S(u), \tag{2.26}$$

$$\langle 0|\bar{q}(0)\gamma^\mu q(0)|A(P,\epsilon_{\lambda=0})\rangle = if_A M_A \epsilon^\mu \int_0^1 du\phi_{A\|}(u). \tag{2.27}$$

Note that the tensor meson cannot be produced by the $V-A$ current, then we pick $n = 1$ in eq. (2.25) and obtain

$$\frac{1}{2}\langle 0|\bar{q}(0)(\gamma^\mu z\cdot\widehat{D} + \not{z}\widehat{D}^\mu)q(0)|T(P,\epsilon_{\lambda=0})\rangle = f_T M_T^2 \epsilon^{\mu\bullet}\int_0^1 du\xi\phi_{T\|}(u). \tag{2.28}$$

From the normalization eq. (2.13), we have $\langle 0|\bar{q}\gamma^\mu\gamma_5 q|^3A_1(P,\epsilon)\rangle = if_{^3A_1}M_{^3A_1}\epsilon^\mu$ which is consistent with the results of [34].

Next, we consider the case of $\Gamma = \sigma_{\mu\nu}(\gamma_5)$, where the leading twist-2 contribution contains contributions of the operators:

$$[\bar{q}(-z)\sigma^{\mu\nu}(\gamma_5)q(z)]_2 = \sum_{n=0}^{\infty}\frac{1}{n!}\bar{q}(0)\Big\{\frac{(z\cdot\widehat{D})^n}{2n+1}\sigma^{\mu\nu} + \frac{n(z\cdot\widehat{D})^{n-1}}{2n+1}\widehat{D}^\mu\sigma^{\bullet\nu} + \frac{n(z\cdot\widehat{D})^{n-1}}{2n+1}\widehat{D}^\nu\sigma^{\mu\bullet}\Big\}(\gamma_5)q(0). \tag{2.29}$$

The sum can be also represented in terms of nonlocal operators:

$$[\bar{q}(-z)\sigma^{\mu\nu}(\gamma_5)q(z)]_2 = \int_0^1 dt\left[\frac{\partial}{\partial z_\mu}\bar{q}(-t^2z)\sigma^{\bullet\nu}(\gamma_5)q(t^2z) + z_\alpha\frac{\partial}{\partial z_\nu}\bar{q}(-t^2z)\sigma^{\mu\alpha}(\gamma_5)q(t^2z)\right]. \tag{2.30}$$

Taking the matrix element between the vacuum and the axial-vector and tensor meson state, we obtain:

$$\begin{aligned}&\langle 0|[\bar{q}(-z)\sigma^{\mu\nu}\gamma_5 q(z)]_2|A(P,\epsilon_{\lambda=\pm1})\rangle \\ &= f_A^\perp\int_0^1 du\Big\{\phi_{A\perp}(u)\Big[\mathcal{S}^{\mu\nu}e^{i\xi pz} + \big((\epsilon^\mu P^\nu - \epsilon^\nu P^\mu) - \mathcal{S}^{\mu\nu}\big)\int_0^1 dte^{i\xi t^2pz}\Big] \\ &\quad + \big(h_{A\|}(u) - \phi_{A\perp}(u)\big)\Big[\mathcal{T}^{\mu\nu}e^{i\xi pz} + \big(\mathcal{U}^{\mu\nu} - \mathcal{T}^{\mu\nu}\big)\int_0^1 dte^{i\xi t^2pz}\Big]\Big\},\end{aligned} \tag{2.31}$$

$$\begin{aligned}&\langle 0|[\bar{q}(-z)\sigma^{\mu\nu}q(z)]_2|T(P,\epsilon_{\lambda=\pm1})\rangle \\ &= if_T^\perp M_T\int_0^1 du\Big\{\phi_{T\perp}(u)\Big[\mathcal{S}'^{\mu\nu}e^{i\xi pz} + \Big(\frac{2(\epsilon^{\mu\bullet}P^\nu - \epsilon^{\nu\bullet}P^\mu)}{pz} - 3\mathcal{S}'^{\mu\nu}\Big)\int_0^1 dte^{i\xi t^2pz}\Big] \\ &\quad + \big(h_{T\|}(u) - \phi_{T\perp}(u)\big)\Big[\mathcal{T}'^{\mu\nu}e^{i\xi pz} + \Big(\frac{2\mathcal{U}'^{\mu\nu}}{pz} - 3\mathcal{T}'^{\mu\nu}\Big)\int_0^1 dte^{i\xi t^2pz}\Big]\Big\},\end{aligned} \tag{2.32}$$

where

$$\mathcal{S}^{\mu\nu} = \frac{1}{2}\left[(\epsilon^\mu P^\nu - \epsilon^\nu P^\mu) - (\epsilon_\perp^\mu z^\nu - \epsilon_\perp^\nu z^\mu)\frac{M_A^2}{2pz}\right],$$
$$\mathcal{T}^{\mu\nu} = \frac{\epsilon z M_A^2}{2(pz)^2}(p^\mu z^\nu - p^\nu z^\mu), \qquad \mathcal{U}^{\mu\nu} = \frac{M_A^2}{pz}(\epsilon^\mu z^\nu - \epsilon^\nu z^\mu),$$
$$\mathcal{S}'^{\mu\nu} = \frac{1}{2pz}\left[(\epsilon^{\mu\bullet} P^\nu - \epsilon^{\nu\bullet} P^\mu) - (\epsilon_\perp^{\mu\bullet} z^\nu - \epsilon_\perp^{\nu\bullet} z^\mu)\frac{M_T^2}{2pz}\right],$$
$$\mathcal{T}'^{\mu\nu} = \frac{\epsilon^{\bullet\bullet} M_T^2}{2(pz)^3}(p^\mu z^\nu - p^\nu z^\mu), \qquad \mathcal{U}'^{\mu\nu} = \frac{M_T^2}{pz}(\epsilon^\mu z^\nu - \epsilon^\nu z^\mu). \tag{2.33}$$

The derivations of eqs. (2.31) and (2.32) are shown in appendix B. In contrast to eqs. (2.20)–(2.22), the twist-2 LCDAs do not disentangle entirely from the higher twists in eqs. (2.31) and (2.32). Taking the product with $\epsilon_{\perp\mu} z_\nu$ and $\epsilon_{\perp\mu\bullet} z_\nu$ in eqs. (2.31) and (2.32), respectively, to obtain,

$$\langle 0|[\bar{q}(-z)\sigma^{\mu\bullet}\epsilon_{\perp\mu}\gamma_5 q(z)]_2|A(P,\epsilon_{\lambda=\pm1})\rangle$$
$$= f_A^\perp \int_0^1 du\phi_{A\perp}(u)\frac{1}{2}(\epsilon\cdot\epsilon_\perp Pz)\left[e^{i\xi pz} + \int_0^1 dt e^{i\xi t^2 pz}\right], \tag{2.34}$$
$$\langle 0|[\bar{q}(-z)\sigma^{\mu\bullet}\epsilon_{\perp\mu\bullet} q(z)]_2|T(P,\epsilon_{\lambda=\pm1})\rangle$$
$$= i f_T^\perp M_T \int_0^1 du\phi_{T\perp}(u)\frac{1}{2}\epsilon^{\mu\bullet}\epsilon_{\perp\mu\bullet}\left[e^{i\xi pz} + \int_0^1 dt e^{i\xi t^2 pz}\right]. \tag{2.35}$$

Then, we use eq. (2.29) and expand the right-hand sides of eqs. (2.34) and (2.35) as:

$$\sum_{n=0}^\infty \frac{1}{n!}\langle 0|\bar{q}(0)\frac{(n+1)(z\cdot\widehat{D})^n}{2n+1}\sigma^{\mu\bullet}\epsilon_{\perp\mu}\gamma_5 q(0)|A(P,\epsilon_{\lambda=\pm1})\rangle$$
$$= f_A^\perp \sum_{n=0}^\infty \frac{i^n}{n!}\int_0^1 du\phi_{A\perp}(u)\frac{1}{2}(\epsilon\cdot\epsilon_\perp Pz)(\xi pz)^n\left[1 + \int_0^1 dt t^{2n}\right], \tag{2.36}$$
$$\sum_{n=0}^\infty \frac{1}{n!}\langle 0|\bar{q}(0)\frac{(n+1)(z\cdot\widehat{D})^n}{2n+1}\sigma^{\mu\bullet}\epsilon_{\perp\mu} q(0)|T(P,\epsilon_{\lambda=\pm1})\rangle$$
$$= i f_T^\perp M_T \sum_{n=0}^\infty \frac{i^n}{n!}\int_0^1 du\phi_{T\perp}(u)\frac{1}{2}\epsilon^{\mu\bullet}\epsilon_{\perp\mu\bullet}(\xi pz)^n\left[1 + \int_0^1 dt t^{2n}\right], \tag{2.37}$$

Picking $n = 0$ in eqs. (2.36) and (2.37), we obtain:

$$\langle 0|\bar{q}(0)\sigma^{\mu\bullet}\epsilon_{\perp\mu}\gamma_5 q(0)|A(P,\epsilon_{\lambda=\pm1})\rangle = f_A^\perp \int_0^1 du\phi_{A\perp}(u)(\epsilon\cdot\epsilon_\perp Pz), \tag{2.38}$$
$$\langle 0|\bar{q}(0)\sigma^{\mu\bullet}\epsilon_{\perp\mu\bullet} q(0)|T(P,\epsilon_{\lambda=\pm1})\rangle = i f_T^\perp M_T \int_0^1 du\phi_{T\perp}(u)\epsilon^{\mu\bullet}\epsilon_{\perp\mu\bullet}. \tag{2.39}$$

It is worth noting that the author of ref. [31] also considered an approach that disentangled the twist-2 LCDAs from the higher twists, in the case of an axial vector meson state ($\Gamma = \sigma^{\mu\nu}\gamma_5$): Besides z_ν, eq. (2.31) has taken the product with a term proportional to

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$(\epsilon_\mu Pz - P_\mu \epsilon z)$. We find this approach equivalent to ours. The derivation is as follows. The term $(\epsilon_\mu Pz - P_\mu \epsilon z)$ can be expanded by using eqs. (2.4) and (2.5) as

$$\begin{aligned}\epsilon_\mu Pz - P_\mu \epsilon z &= p_\mu \frac{ez}{pz} Pz - z_\mu \frac{ezM_A^2}{2(pz)^2} Pz + \epsilon_{\perp\mu} Pz - p_\mu ez - z_\mu \frac{ezM_A^2}{2pz} \\ &= -z_\mu \frac{ezM_A^2}{pz} + \epsilon_{\perp\mu} Pz. \end{aligned} \tag{2.40}$$

The first term of last line has no contribution to the result because $\sigma^{\mu\nu}$ is antisymmetric.

3 General formulism in LFQM

3.1 Framework

A meson bound state, consisting of a quark q_1 and an antiquark $\bar{q}_2$ with total momentum P and spin J, can be written as (see, for example [26]):

$$\begin{aligned}|M(P,^{2S+1}L_J, J_z)\rangle = \int \{d^3k_1\}\{d^3k_2\}\ 2(2\pi)^3\delta^3(\tilde{P} - \tilde{k}_1 - \tilde{k}_2) \\ \times \sum_{\lambda_1,\lambda_2} \Psi^{JJ_z}_{\rm LS}(\tilde{k}_1, \tilde{k}_2, \lambda_1, \lambda_2)\ |q_1(k_1,\lambda_1)\bar{q}_2(k_2,\lambda_2)\rangle,\end{aligned} \tag{3.1}$$

where k_1 and k_2 are the on-mass-shell light-front momenta,

$$\tilde{k} = (k^+, k_\perp), \quad k_\perp = (k^1, k^2), \quad k^- = \frac{m_q^2 + k_\perp^2}{k^+}, \tag{3.2}$$

and

$$\begin{aligned}\{d^3k\} &\equiv \frac{dk^+ d^2k_\perp}{2(2\pi)^3}, \\ |q(k_1,\lambda_1)\bar{q}(k_2,\lambda_2)\rangle &= b^\dagger_{\lambda_1}(k_1) d^\dagger_{\lambda_2}(k_2)|0\rangle, \\ \{b_{\lambda'}(k'), b^\dagger_\lambda(k)\} &= \{d_{\lambda'}(k'), d^\dagger_\lambda(k)\} = 2(2\pi)^3\ \delta^3(\tilde{k}' - \tilde{k})\ \delta_{\lambda'\lambda}.\end{aligned} \tag{3.3}$$

In terms of the light-front relative momentum variables $(u, \kappa_\perp)$ defined by

$$\begin{aligned} k_1^+ &= (1-u)P^+, & k_2^+ &= uP^+, \\ k_{1\perp} &= (1-u)P_\perp + \kappa_\perp, & k_{2\perp} &= uP_\perp - \kappa_\perp. \end{aligned} \tag{3.4}$$

The relative momentum in $\hat{z}$ direction κ_z can be written as

$$\kappa_z = \frac{uM_0}{2} - \frac{m_2^2 + \kappa_\perp^2}{2uM_0}. \tag{3.5}$$

The momentum-space wave-function $\Psi^{JJ_z}_{\rm LS}$ for a $^{2S+1}L_J$ meson can be expressed as

$$\Psi^{JJ_z}_{\rm LS}(\tilde{k}_1, \tilde{k}_2, \lambda_1, \lambda_2) = \frac{1}{\sqrt{N_c}} \langle LS; L_z S_z | LS; JJ_z\rangle R^{SS_z}_{\lambda_1\lambda_2}(u, \kappa_\perp)\ \varphi_{LL_z}(u, \kappa_\perp), \tag{3.6}$$

where $\varphi_{LL_z}(u,\kappa_\perp)$ describes the momentum distribution of the constituent quarks in the bound state with the orbital angular momentum L, $\langle LS;L_zS_z|LS;JJ_z\rangle$ is the corresponding Clebsch-Gordan coefficient and $R^{SS_z}_{\lambda_1\lambda_2}$ constructs a state of definite spin (S,S_z) out of light-front helicity (λ_1,λ_2) eigenstates. Explicitly,

$$R^{SS_z}_{\lambda_1\lambda_2}(u,\kappa_\perp)=\sum_{s_1,s_2}\langle\lambda_1|\mathcal{R}^\dagger_M(1-u,\kappa_\perp,m_1)|s_1\rangle\langle\lambda_2|\mathcal{R}^\dagger_M(u,-\kappa_\perp,m_2)|s_2\rangle\langle\frac{1}{2}\,\frac{1}{2};s_1s_2|\frac{1}{2}\frac{1}{2};SS_z\rangle, \tag{3.7}$$

where $|s_i\rangle$ are the usual Pauli spinors, and $\mathcal{R}_M$ is the Melosh transformation operator [25]:

$$\langle s|\mathcal{R}_M(u_i,\kappa_\perp,m_i)|\lambda\rangle\;=\;\frac{m_i+u_iM_0+i\vec{\sigma}_{s\lambda}\cdot\vec{\kappa}_\perp\times\vec{n}}{\sqrt{(m_i+u_iM_0)^2+\kappa_\perp^2}}, \tag{3.8}$$

with $u_1=1-u$, $u_2=u$, and $\vec{n}=(0,0,1)$ is a unit vector in the $\hat{z}$-direction. In addition,

$$\begin{aligned}M_0^2\;&=\;(e_1+e_2)^2=\frac{m_1^2+\kappa_\perp^2}{u_1}+\frac{m_2^2+\kappa_\perp^2}{u_2},\\ e_i\;&=\;\sqrt{m_i^2+\kappa_\perp^2+\kappa_z^2}.\end{aligned} \tag{3.9}$$

where M_0 is the invariant mass of $q\bar{q}$ and generally different from the mass M of meson which satisfies $M^2=P^2$. This is due to the fact that the meson, quark and anti-quark cannot be simultaneously on-shell. We normalize the meson state as

$$\langle M(P',J',J'_z)|M(P,J,J_z)\rangle=2(2\pi)^3P^+\delta^3(\tilde{P}'-\tilde{P})\delta_{J'J}\delta_{J'_zJ_z}\,, \tag{3.10}$$

in order that:

$$\int\frac{du\,d^2\kappa_\perp}{2(2\pi)^3}\;\varphi'^*_{L'L'_z}(u,\kappa_\perp)\varphi_{LL_z}(u,\kappa_\perp)=\delta_{L',L}\;\delta_{L'_z,L_z}. \tag{3.11}$$

Explicitly, we have:

$$\varphi_{1L_z}=\kappa_{L_z}\varphi_p, \tag{3.12}$$

where $\kappa_{L_z=\pm1}=\mp(\kappa_{\perp x}\pm i\kappa_{\perp y})/\sqrt{2}$, $\kappa_{L_z=0}=\kappa_z$ are proportional to the spherical harmonics Y_{1L_z} in momentum space, φ_p is the distribution amplitude of p-wave meson. In general, for any function $F(\vec{\kappa})$, $\varphi_p(u)$ has the form of:

$$\varphi_p(u)=N\sqrt{\frac{d\kappa_z}{du}}F(\vec{\kappa}), \tag{3.13}$$

where the normalization factor N is determined from eq. (3.11).

In the case of a p-wave meson state, it is more convenient to use the covariant form of $R^{SS_z}_{\lambda_1\lambda_2}$ [25, 28, 36]:

$$\begin{aligned}\langle 1S;L_zS_z|1S;JJ_z\rangle\,k_{L_z}\,R^{SS_z}_{\lambda_1\lambda_2}(u,\kappa_\perp)\;=\;&\frac{\sqrt{k_1^+k_2^+}}{\sqrt{2}\;\widetilde{M}_0(M_0+m_1+m_2)}\\ &\times\bar{u}(k_1,\lambda_1)(\bar{\not{P}}+M_0)\Gamma_{2S+1P_J}v(k_2,\lambda_2),\end{aligned} \tag{3.14}$$

where

$$\widetilde{M}_0 \equiv \sqrt{M_0^2-(m_1-m_2)^2}, \qquad \bar{P}\equiv k_1+k_2,$$
$$\bar{u}(k,\lambda)u(k,\lambda')=\frac{2m}{k^+}\delta_{\lambda,\lambda'}, \qquad \sum_\lambda u(k,\lambda)\bar{u}(k,\lambda)=\frac{\not{k}+m}{k^+},$$
$$\bar{v}(k,\lambda)v(k,\lambda')=-\frac{2m}{k^+}\delta_{\lambda,\lambda'}, \qquad \sum_\lambda v(k,\lambda)\bar{v}(k,\lambda)=\frac{\not{k}-m}{k^+}. \tag{3.15}$$

For the scalar, axial-vector, and tensor mesons, we have:

$$\begin{aligned}
\Gamma_{^3P_0} &= \frac{1}{\sqrt{3}}\left(\not{K}-\frac{K\cdot\bar{P}}{M_0}\right),\\
\Gamma_{^1P_1} &= -\epsilon\cdot K\gamma_5,\\
\Gamma_{^3P_1} &= \frac{1}{\sqrt{2}}\left(\left(\not{K}-\frac{K\cdot\bar{P}}{M_0}\right)\not{\epsilon}-\epsilon\cdot K\right)\gamma_5,\\
\Gamma_{^3P_2} &= \epsilon_{\mu\nu}\gamma^\mu(-K^\nu),
\end{aligned} \tag{3.16}$$

where $K\equiv(k_2-k_1)/2$ and:

$$\begin{aligned}
\epsilon^\mu_{\lambda=\pm1} &= \left[\frac{2}{P^+}\vec{\epsilon}_\perp(\pm1)\cdot\vec{P}_\perp,\,0,\,\vec{\epsilon}_\perp(\pm1)\right],\\
\vec{\epsilon}_\perp(\pm1) &= \mp(1,\pm i)/\sqrt{2},\\
\epsilon^\mu_{\lambda=0} &= \frac{1}{M_0}\left(\frac{-M_0^2+P_\perp^2}{P^+},P^+,P_\perp\right).
\end{aligned} \tag{3.17}$$

Note that the polarization tensor of a tensor meson satisfies the relations: $\epsilon_{\mu\nu}=\epsilon_{\nu\mu}$ and $\epsilon_{\mu\nu}\bar{P}^\mu=\epsilon^\mu_\mu=0$. Eqs. (3.14) and (3.16) can be further reduced by the applications of equations of motion on spinors:

$$\langle 1S;L_zS_z|1S;JJ_z\rangle\, k_{L_z}\, R^{SS_z}_{\lambda_1\lambda_2}(u,\kappa_\perp)=\frac{\sqrt{k_1^+k_2^+}}{\sqrt{2}\,\widetilde{M}_0}\bar{u}(k_1,\lambda_1)\Gamma'_{^{2S+1}P_J}v(k_2,\lambda_2), \tag{3.18}$$

where

$$\begin{aligned}
\Gamma'_{^3P_0} &= -\frac{\widetilde{M}_0^2}{2\sqrt{3}M_0},\\
\Gamma'_{^1P_1} &= -\epsilon\cdot K\gamma_5,\\
\Gamma'_{^3P_1} &= \frac{-1}{2\sqrt{2}M_0}\left(\not{\epsilon}\widetilde{M}_0^2-2\epsilon\cdot K(m_1-m_2)\right)\gamma_5,\\
\Gamma'_{^3P_2} &= \epsilon_{\mu\nu}\left(\gamma^\mu+\frac{2K^\mu}{M_0+m_1+m_2}\right)(-K^\nu).
\end{aligned} \tag{3.19}$$

3.2 Analysis of Leading twist LCDAs

Next, the matrix elements of eqs. (2.26), (2.27), (2.28), (2.38), and (2.39) will be calculated within LFQM, and the relevant leading twist LCDAs are extracted. For the scalar meson

state, we substitute eqs. (3.1), (3.6), and (3.18) into eq. (2.26) to obtain:

$$\begin{aligned}\langle 0|\bar{q}_2\gamma^\mu q_1|S(P)\rangle &= N_c\int\{d^3k_1\}\sum_{\lambda_1,\lambda_2}\Psi_{\rm LS}^{JJ_z}(k_1,k_2,\lambda_1,\lambda_2)\langle 0|\bar{q}_2\gamma^\mu q_1|q_1\bar{q}_2\rangle\\ &= -\sqrt{N_c}\int\{d^3k_1\}\frac{\sqrt{k_1^+k_2^+}}{\sqrt{2}\ \widetilde{M}_0}\varphi_p{\rm Tr}\left[\gamma^\mu\left(\frac{\not{k}_1+m_1}{k_1^+}\right)\frac{\widetilde{M}_0^2}{2\sqrt{3}M_0}\left(\frac{-\not{k}_2+m_2}{k_2^+}\right)\right]\\ &= f_S P^\mu\int du\phi(u).\end{aligned}\tag{3.20}$$

For the "good" component, $\mu=+$, the leading twist LCDA ϕ_S can be extracted as:

$$\phi_S(u)=\frac{\sqrt{2N_c}}{f_S}\int\frac{d^2\kappa_\perp}{2(2\pi)^3}\frac{[(1-u)m_2-um_1]\widetilde{M}_0}{\sqrt{3u(1-u)}M_0}\varphi_p(u,\kappa_\perp).\tag{3.21}$$

In the quarkonium case $m_1=m_2=m$, eq. (3.21) can be further reduced as:

$$\phi_S(u)=\frac{\sqrt{2}}{f_S}\int\frac{d^2\kappa_\perp}{2(2\pi)^3}\frac{(1-2u)m}{\sqrt{u(1-u)}}\varphi_p(u,\kappa_\perp).\tag{3.22}$$

A similar process can be used for the axial vector and tensor mesons which correspond to eqs. (2.27), (2.38) and (2.28), (2.39), respectively, and the leading twist LCDAs are extracted as

$$\phi_{^3A_1\parallel}(u)=\frac{2\sqrt{3}}{f_{^3A_1}}\int\frac{d^2\kappa_\perp}{2(2\pi)^3}\frac{\kappa_\perp^2}{\sqrt{u(1-u)}M_0}\varphi_p(u,\kappa_\perp),\tag{3.23}$$

$$\phi_{^1A_1\parallel}(u)=\frac{\sqrt{6}}{f_{^1A_1}}\int\frac{d^2\kappa_\perp}{2(2\pi)^3}\frac{(1-2u)m}{\sqrt{u(1-u)}}\varphi_p(u,\kappa_\perp),\tag{3.24}$$

$$\phi_{T\parallel}(u)=\frac{\sqrt{6}}{f_T}\int\frac{d^2\kappa_\perp}{2(2\pi)^3}\frac{(1-2u)}{\sqrt{u(1-u)}}\left[M_0-m-\frac{\kappa_\perp^2}{M_0+2m}\right]\varphi_p(u,\kappa_\perp),\tag{3.25}$$

$$\phi_{^3A_1\perp}(u)=\frac{\sqrt{3}}{f_{^3A_1}^\perp}\int\frac{d^2\kappa_\perp}{2(2\pi)^3}\frac{(1-2u)m}{\sqrt{u(1-u)}}\varphi_p(u,\kappa_\perp),\tag{3.26}$$

$$\phi_{^1A_1\perp}(u)=\frac{\sqrt{6}}{f_{^1A_1}^\perp}\int\frac{d^2\kappa_\perp}{2(2\pi)^3}\frac{\kappa_\perp^2}{\sqrt{u(1-u)}M_0}\varphi_p(u,\kappa_\perp),\tag{3.27}$$

$$\phi_{T\perp}(u)=\frac{\sqrt{6}}{f_T^\perp}\int\frac{d^2\kappa_\perp}{2(2\pi)^3}\frac{(1-2u)}{\sqrt{u(1-u)}}\left[m+\frac{2\kappa_\perp^2}{M_0+2m}\right]\varphi_p(u,\kappa_\perp).\tag{3.28}$$

From the normalization eq. (2.13), some relations among the constants f_Ms could be easily obtained as:

$$\sqrt{3}f_S=f_{^1A_1}=\sqrt{2}f_{^3A_1}^\perp\equiv f_{\rm odd},\quad\text{and}\quad\frac{f_{^3A_1}}{\sqrt{2}}=f_{^1A_1}^\perp\equiv f_{\rm even},\tag{3.29}$$

then, the relations among the relevant LCDAs are:

$$\phi_S=\phi_{^1A_1\parallel}=\phi_{^3A_1\perp}\equiv\phi_{\rm odd},\quad\text{and}\quad\phi_{^3A_1\parallel}=\phi_{^1A_1\perp}\equiv\phi_{\rm even},\tag{3.30}$$

where the subscript "odd (even)" means the odd (even) function of u and

$$\phi_{\rm odd}(u) = \frac{\sqrt{6}}{f_{\rm odd}} \int \frac{d^2\kappa_\perp}{2(2\pi)^3} \frac{(1-2u)m}{\sqrt{u(1-u)}} \varphi_p(u,\kappa_\perp), \tag{3.31}$$

$$\phi_{\rm even}(u) = \frac{\sqrt{6}}{f_{\rm even}} \int \frac{d^2\kappa_\perp}{2(2\pi)^3} \frac{\kappa_\perp^2}{\sqrt{u(1-u)}M_0} \varphi_p(u,\kappa_\perp) \tag{3.32}$$

Note that eqs. (3.29) and (3.30) are independent of the form of φ_p. Furthermore, in the nonrelativistic situation, the momenta $\kappa_{z,\perp}$ are much smaller than quark mass m, and M_0 can be reduced to approximately $2m$. Thus, we have:

$$f_{\rm odd} \simeq f_T \simeq f_T^\perp, \quad \phi_{\rm odd} \simeq \phi_{T\parallel} \simeq \phi_{T\perp}. \tag{3.33}$$

and

$$\phi_{\rm odd}(u) \simeq \frac{\sqrt{6}}{f_{\rm odd}} \int \frac{d^2\kappa_\perp}{2(2\pi)^3} \frac{(1-2u)}{\sqrt{u(1-u)}} \frac{M_0}{2} \varphi_p(u,\kappa_\perp). \tag{3.34}$$

From eqs. (3.32) and (3.34), one can obtain:

$$f_{\rm odd} \simeq f_{\rm even}, \tag{3.35}$$

and relate the ξ-moments of the $\phi_{\rm even}$ to those of the $\phi_{\rm odd}$ as

$$\langle \xi^n \rangle_{\phi_{\rm even}} \simeq \frac{\langle \xi^{n+1} \rangle_{\phi_{\rm odd}}}{n+1}, \tag{3.36}$$

with the function $F = F(|\vec{\kappa}|)$. The derivations of eqs. (3.35) and (3.36) are shown in appendix C. The above results are consistent with those of [6] in the nonrelativistic approximation.[1]

Next, we choose a Gaussian-like wave function, as shown in [28]:

$$\varphi_p(u,\kappa_\perp) = \frac{4\sqrt{2}}{\beta}\left(\frac{\pi}{\beta^2}\right)^{3/4} \sqrt{\frac{d\kappa_z}{du}} \exp\left(-\frac{|\vec{\kappa}|^2}{2\beta^2}\right), \tag{3.37}$$

for further calculations. In eqs. (3.31) and (3.32), the $\kappa_\perp$ integrations can be performed as follows:

$$\phi_{\rm odd}(u) = \frac{\sqrt{6}(1-2u)m}{f_{\rm odd}} \left(\frac{2}{\pi}\right)^{5/4} e^d \Gamma\left[\frac{5}{4}, w\right], \tag{3.38}$$

$$\phi_{\rm even}(u) = \frac{\sqrt{3}u(1-u)\beta}{f_{\rm even}} \left(\frac{2}{\pi}\right)^{5/4} e^d \left\{ w\Gamma\left[-\frac{1}{4}, w\right] + 3\Gamma\left[\frac{3}{4}, w\right] \right\}, \tag{3.39}$$

[1] For the tensor mesons, ref. [6] has the relation $f_T = \sqrt{\frac{2}{3}} f_T^\perp$. The additional $\sqrt{\frac{2}{3}}$ factor is the Clebsch-Gordan coefficient of the polarization tensor $\epsilon_{\mu\nu}$ for the tensor meson state $T(P, \epsilon_{\lambda=0})$. This distinction is from the different definition of $\phi_{T\parallel}$: There is a $\epsilon_{\mu\nu}$ in both hand sides of eq. (2.11). By contrast, in the upper part of eq. (4) of ref. [6], $\epsilon_{\mu\nu}$ appears only in the left hand side.

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where $w = d/[4u(1-u)]$, $d = m^2/2\beta^2$ and:

$$\Gamma[a,w] = \int_w^\infty t^{a-1}e^{-t}dt$$

is the incomplete Gamma function. The incomplete gamma function may be expressed quite elegantly in terms of the confluent hypergeometric function:

$$\Gamma[a,w] = \Gamma[a] - a^{-1}w^a \times \ {}_1F_1(a;a+1;-w), \tag{3.40}$$

where

$${}_iF_j(a_1,a_2,\ldots,a_i;a_1',a_2',\ldots,a_j';w) = \sum_{n=0}^{\infty}\frac{(a_1)_n(a_2)_n\ldots(a_i)_n}{(a_1')_n(a_2')_n\ldots(a_j')_n}\frac{w^n}{n!}, \tag{3.41}$$

and $(a)_n = (a+n-1)!/(a-1)!$ is the Pochhammer symbol. However, the $\kappa_\perp$ integrals in eqs. (3.25) and (3.28) cannot be analytically performed. The crux is the term proportional to $1/M_0 + 2m$. This term may be rewritten and expanded as:

$$\frac{\kappa_\perp^2}{M_0+2m} = \frac{\kappa_\perp^2}{4m\left(1+\frac{M_0-2m}{4m}\right)} = \frac{\kappa_\perp^2}{4m}\left[1-\left(\frac{M_0-2m}{4m}\right)+\ldots\right]. \tag{3.42}$$

We only consider the first two terms in the square bracket of eq. (3.42) because $M_0 - 2m$ goes to zero in the non-relativistic approximation. Then, the approximate form of, for example, $\phi_{T\parallel}(u)$ which containing the first one term and first two terms in the square bracket of eq. (3.42) are defined as:

$$\phi_{T\parallel}^{(1)}(u) = \frac{\sqrt{6}}{f_T}\int\frac{d^2\kappa_\perp}{2(2\pi)^3}\frac{(1-2u)}{\sqrt{u(1-u)}}\left[M_0 - m - \frac{\kappa_\perp^2}{4m}\right]\varphi_p(u,\kappa_\perp), \tag{3.43}$$

$$\phi_{T\parallel}^{(2)}(u) = \frac{\sqrt{6}}{f_T}\int\frac{d^2\kappa_\perp}{2(2\pi)^3}\frac{(1-2u)}{\sqrt{u(1-u)}}\left[M_0 - m - \frac{\kappa_\perp^2}{4m}\left(1-\frac{M_0-2m}{4m}\right)\right]\varphi_p(u,\kappa_\perp), \tag{3.44}$$

respectively. The $\kappa_\perp$ integrals can be performed as:

$$\phi_{T\parallel}^{(1)}(u) = \frac{\sqrt{6}(1-2u)m}{f_T}\left(\frac{2}{\pi}\right)^{5/4}e^d\left\{-\frac{3}{4}\Gamma\left[\frac{5}{4},w\right] - \frac{1}{4w}\Gamma\left[\frac{9}{4},w\right] + \frac{4\beta}{\sqrt{2}m}\Gamma\left[\frac{7}{4},w\right]\right\}, \tag{3.45}$$

$$\begin{aligned}\phi_{T\parallel}^{(2)}(u) = \frac{\sqrt{6}(1-2u)m}{f_T}\left(\frac{2}{\pi}\right)^{5/4}e^d\Bigg\{&-\frac{5}{8}\Gamma\left[\frac{5}{4},w\right] - \frac{3}{8w}\Gamma\left[\frac{9}{4},w\right] + \frac{15\beta}{4\sqrt{2}m}\Gamma\left[\frac{7}{4},w\right]\\ &+\frac{\beta}{4\sqrt{2}wm}\Gamma\left[\frac{11}{4},w\right]\Bigg\}.\end{aligned} \tag{3.46}$$

We can find that the both curves of $\phi_{T\parallel}^{(1)}(u)$ and $\phi_{T\parallel}^{(2)}(u)$ are almost overlap each other by using the parameters in table 1 (See section 4). Therefore we only consider the first term in the square bracket of eq. (3.42) and take the approximation $\phi_{T\parallel}(u) \simeq \phi_{T\parallel}^{(1)}(u)$. Finally the form of $\phi_{T\perp}(u)$ is:

$$\phi_{T\perp}(u) \simeq \frac{\sqrt{6}(1-2u)m}{2f_T^\perp}\left(\frac{2}{\pi}\right)^{5/4}e^d\left\{\Gamma\left[\frac{5}{4},r\right] + \frac{1}{w}\Gamma\left[\frac{9}{4},w\right]\right\}. \tag{3.47}$$

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In addition, the ξ-moments of eqs. (3.38), (3.39), (3.45), and (3.47) can be analytically expressed as:

$$\langle\xi^{2l+1}\rangle_{\phi_{\rm odd}} = A_{\rm odd}\left\{\frac{d^{5/4}\Gamma\left[\frac{-5}{4}\right]}{\Gamma\left[\frac{5}{4}+l\right]}\ {}_1F_1\left(-\frac{1}{4}-l;\frac{9}{4};-d\right)+\frac{\Gamma\left[\frac{5}{4}\right]}{\Gamma\left[\frac{5}{2}+l\right]}\ {}_1F_1\left(-\frac{3}{2}-l;-\frac{1}{4};-d\right)\right\}, \tag{3.48}$$

$$\begin{aligned}\langle\xi^{2l}\rangle_{\phi_{\rm even}} = A_{\rm even}\Bigg\{&\frac{d^{3/4}\Gamma\left[\frac{1}{4}\right]}{\Gamma\left[\frac{7}{4}+l\right]}\left[\ {}_1F_1\left(-\frac{3}{4}-l;\frac{3}{4};-d\right)-\ {}_2F_2\left(-\frac{3}{4}-l,\frac{3}{4};-\frac{1}{4},\frac{7}{4};-d\right)\right]\\ &-\frac{3\Gamma\left[\frac{-1}{4}\right]}{4\Gamma\left[\frac{5}{2}+l\right]}\ {}_1F_1\left(-\frac{3}{2}-l;-\frac{3}{4};-d\right)\Bigg\},\end{aligned} \tag{3.49}$$

$$\begin{aligned}\langle\xi^{2l+1}\rangle_{\phi_T} \simeq A_T\Bigg\{&\frac{\sqrt{2}d^{5/4}\Gamma\left[\frac{-5}{4}\right]}{\Gamma\left[\frac{5}{4}+l\right]}\left[3\ {}_1F_1\left(-\frac{1}{4}-l;\frac{9}{4};-d+\right)+\frac{5}{9}\ {}_2F_2\left(-\frac{1}{4}-l,\frac{9}{4};\frac{5}{4},\frac{13}{4};-d\right)\right]\\ &+\frac{\sqrt{2}\Gamma\left[\frac{5}{4}\right]}{\Gamma\left[\frac{7}{2}+l\right]}\left[\frac{5}{4d}\ {}_1F_1\left(-\frac{3}{2}-l;-\frac{5}{4};-d\right)+(9+4l)\ {}_1F_1\left(-\frac{3}{2}-l;-\frac{1}{4};-d\right)\right]\\ &-16\frac{\beta}{m}\left[\frac{\Gamma\left[\frac{7}{4}\right]}{\Gamma\left[\frac{5}{2}+l\right]}\ {}_1F_1\left(-\frac{3}{2}-l;-\frac{3}{4};-d\right)+\frac{d^{7/4}\Gamma\left[-\frac{7}{4}\right]}{\Gamma\left[\frac{3}{4}+l\right]}\ {}_1F_1\left(\frac{1}{4}-l;\frac{11}{4};-d\right)\right]\Bigg\},\end{aligned} \tag{3.50}$$

$$\begin{aligned}\langle\xi^{2l+1}\rangle_{\phi_{T\perp}} \simeq A_{T\perp}\Bigg\{&\frac{d^{5/4}\Gamma\left[-\frac{5}{4}\right]}{\Gamma\left[\frac{5}{4}+l\right]}\left[\ {}_1F_1\left(-\frac{1}{4}-l;\frac{9}{4};-d\right)+\frac{5}{9}\ {}_2F_2\left(-\frac{1}{4}-l,\frac{9}{4};\frac{5}{4},\frac{13}{4};-d\right)\right]\\ &+\frac{2\Gamma\left[\frac{9}{4}\right]}{d\Gamma\left[\frac{5}{2}+l\right]}\ {}_1F_1\left(-\frac{3}{2}-l;-\frac{5}{4};-d\right)-\frac{4+2l}{5+2l}\ {}_1F_1\left(-\frac{5}{2}-l;-\frac{5}{4};-d\right)\Bigg\},\end{aligned} \tag{3.51}$$

where

$$A_{\rm odd} = \frac{\sqrt{6}m}{2f_{\rm odd}}\left(\frac{2}{\pi}\right)^{5/4} e^d\Gamma\left[\frac{3}{2}+l\right], \qquad A_{\rm even} = \frac{\sqrt{3}\beta}{8f_{\rm even}}\left(\frac{2}{\pi}\right)^{5/4} e^d\Gamma\left[\frac{1}{2}+l\right],$$
$$A_T = -\frac{\sqrt{3}m}{8f_T}\left(\frac{2}{\pi}\right)^{5/4} e^d\Gamma\left[\frac{3}{2}+l\right], \qquad A_{T\perp} = \sqrt{\frac{3}{8}}\frac{m}{f_{T\perp}}\left(\frac{2}{\pi}\right)^{5/4} e^d\Gamma\left[\frac{3}{2}+l\right],$$

and l is a non-negative integer. The derivations of eqs. (3.48)–(3.51) used the formula:

$${}_1F_1(a;b;c) = \frac{b-1}{c}\left[\ {}_1F_1(a;b-1;c)-\ {}_1F_1(a-1;b-1;c)\right], \tag{3.52}$$

which is easily checked from the definition of the confluent hypergeometric function eq. (3.41).

4 Numerical results and discussions

In this section, the LCDs, constants fs, and ξ-moments are estimated. Prior to numerical calculations, the parameters m and β, which appeared in the wave function, must be determined firstly. We consider the Hamiltonian of the p-wave heavy quarkonium state as:

$$H = 2\sqrt{m^2+\vec{\kappa}^2}+br-\frac{4\alpha_s}{3r}+g_1S\cdot L+g_2S_{12}+g_3s_1\cdot s_2, \tag{4.1}$$

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$b(\mathrm{GeV}^2)$	α_s	$m_c(\mathrm{GeV})$	$\beta_{cc}(\mathrm{GeV})$	$m_b(\mathrm{GeV})$	$\beta_{bb}(\mathrm{GeV})$
0.18 ± 0.02	0.36	$1.38^{-0.03}_{+0.04}$	$0.489^{+0.015}_{-0.014}$	4.76 ∓ 0.02	$0.791^{+0.022}_{-0.023}$
0.18	0.36 ± 0.04	1.38 ± 0.01	0.489 ± 0.007	4.76 ± 0.02	$0.791^{+0.022}_{-0.021}$

Table 1. The connections among the parameters b, α_s and m, β of the p-wave heavy quarkonium states.

where br $(-\frac{4\alpha_s}{3r})$ is the linear (Coulomb) potential, $S_{12} = (3s_1 \cdot \hat{r}\ s_2 \cdot \hat{r} - s_1 \cdot s_2)$ is the tensor force operator, and $g_{1,2,3}$ are the functions of the relevant interquark potentials (the details are shown in, for example, [37, 38]). In this way, the mass difference between the spin-single ground state $M(^1P_1)$ and the spin-weighted average of the triplet states $M(^3P_J) \equiv [M(^3P_0) + 3M(^3P_1) + 5M(^3P_2)]/9$ only has the contribution which comes from the spin-spin interaction.[2] Experimentally this hyperfine splitting is less than 1 Mev [16] in charmonium sector, and can be neglected here. Then, we can use the mass $M(^3P_J)$ and its variational principle for the Hamiltonian eq. (4.1) in order to determine parameters m and β. In the process, the conjugate coordinate wave function of eq. (3.37)

$$\tilde{\varphi}_m^{1P}(r) = \sqrt{\frac{8}{3}}\frac{\beta^{3/2}}{\pi^{1/4}}\ \beta r\ \exp\left(-\frac{\beta^2 r^2}{2}\right) Y_{1m}(\theta, \phi), \tag{4.2}$$

is necessary. The values of the additional parameters b and α_s come from literature [39]:

$$b = 0.18\ \mathrm{GeV}^2, \qquad \alpha_s = 0.36, \tag{4.3}$$

for the heavy quarkonium states. We individually vary b and α_s to realize how they connect to m and β. The results are shown in table 1. We find the parameters m and β insensitively depend on b and α_s.

Next, we use the center values of the parameters in table 1 to calculate and plot the LCDAs in eqs. (3.38), (3.39), (3.45), and (3.47). The results are as follows:

$$\begin{aligned} f_{\mathrm{odd}} &= 0.0884\ \mathrm{GeV}, & f_{\mathrm{even}} &= 0.109\ \mathrm{GeV}, \\ f_{T\parallel} &= 0.124\ \mathrm{GeV}, & f_{T\perp} &= 0.0978\ \mathrm{GeV}, \end{aligned} \tag{4.4}$$

for the charmonium states, and

$$\begin{aligned} f_{\mathrm{odd}} &= 0.0674\ \mathrm{GeV}, & f_{\mathrm{even}} &= 0.0716\ \mathrm{GeV}, \\ f_{T\parallel} &= 0.0750\ \mathrm{GeV}, & f_{T\perp} &= 0.0692\ \mathrm{GeV}, \end{aligned} \tag{4.5}$$

for the bottomonium state. The curves of LCDAs are shown in figures 1 and 2. These results are consistent with eqs. (3.33) and (3.35). In addition, the curve of $\phi^{bb}(\xi)$ in figure 2, in which ξ is peaked around zero, was sharper than that of $\phi^{cc}(\xi)$. This meant that the momentum fraction u in the bottomonium state is more centered on 1/2 than in the charmonium state, which is reasonable, as the mass of b quark is larger than that of c quark. A similar situation exists for the odd functions in figure 1.

[2]The calculations of the expectation values of the fourth and fifth terms for the Hamiltonian eq. (4.1) can refer to the appendix of ref. [40].

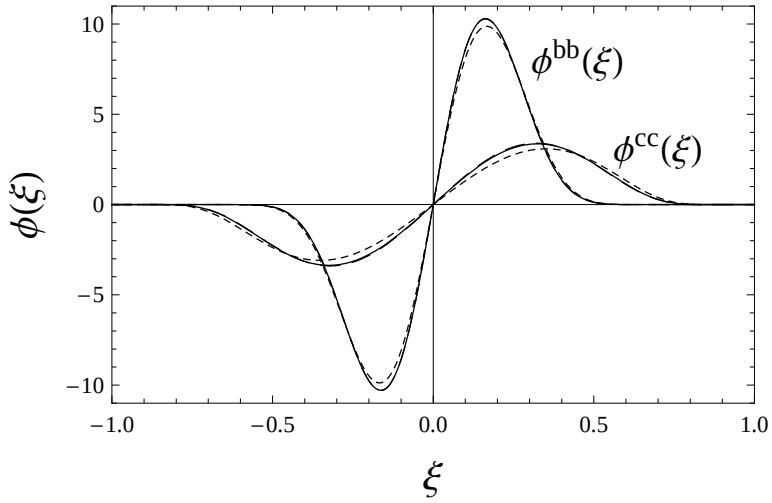

Figure 1. The LCDAs $\phi_{\rm odd}(\xi)$ (solid line), $\phi_{T\parallel}(\xi)$ (dashed line), and $\phi_{T\perp}(\xi)$ (long dashed line) of the charmonium and bottomonium states. The solid line completely overlaps the long dashed line.

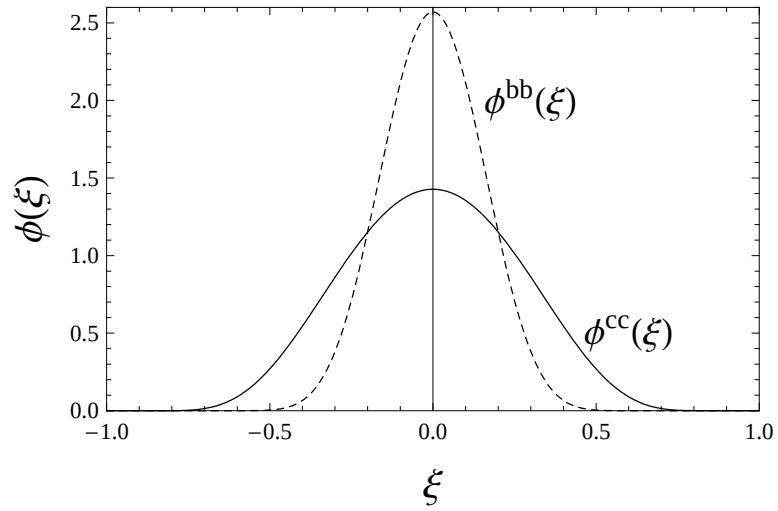

Figure 2. The LCDAs $\phi_{\rm even}(\xi)$ of the charmonium (solid line) and bottomonium (dashed line) states.

Finally we show the LCDAs in terms of the ξ-moments. eqs. (3.48), (3.50), (3.51), and (3.49) are calculated for $l = 3, 5, 7$ and $l = 2, 4, 6$, respectively. The results, which compare with the other theoretical evaluations, are as shown in tables 2 and 3. In table 2, refs. [6] used the QCD sum rules with the non-relativistic wave functions. The authors of ref. [41] calculated in the framework of the Buchmuller-Tye potential model, and found their results are in agreement with experiments which included the leptonic widths and hyperfine splittings. The authors of [42] calculated in the Cornell potential. For the charmonium sector, our results are not only consistent with those of [6, 41, 42], but also conform to eq. (3.36). In the framework of NRQCD, the authors of ref. [6] related the relative velocity

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moment	this work	[6][†‡]	[41]	[42]
$\langle\xi^3\rangle_{\phi_{\rm odd}}$	0.190	0.18 ± 0.03	0.18	0.16
$\langle\xi^5\rangle_{\phi_{\rm odd}}$	0.0507	0.050 ± 0.010	0.047	0.040
$\langle\xi^7\rangle_{\phi_{\rm odd}}$	0.0164	0.017 ± 0.004	0.016	0.013
$\langle\xi^3\rangle_{\phi_T}$	0.206			
$\langle\xi^5\rangle_{\phi_T}$	0.0583			
$\langle\xi^7\rangle_{\phi_T}$	0.0198			
$\langle\xi^3\rangle_{\phi_{T\perp}}$	0.188			
$\langle\xi^5\rangle_{\phi_{T\perp}}$	0.0498			
$\langle\xi^7\rangle_{\phi_{T\perp}}$	0.0160			
$\langle\xi^2\rangle_{\phi_{\rm even}}$	0.0662	0.06 ± 0.01		
$\langle\xi^4\rangle_{\phi_{\rm even}}$	0.0110	0.010 ± 0.002		
$\langle\xi^6\rangle_{\phi_{\rm even}}$	0.00261	0.0024 ± 0.0006		

Table 2. The ξ-moments for the p-wave charmonium states. ([†] $\langle\xi^i\rangle_{\phi_{\rm odd}} = \langle\xi^i\rangle_{h_c}$. [‡] $\langle\xi^i\rangle_{\phi_{\rm even}} = \langle\xi^{i+1}\rangle_{h_c}/(i+1)$)

$\langle\xi^3\rangle_{\phi_{\rm odd}}$	0.0666	$\langle\xi^3\rangle_{\phi_T}$	0.0691	$\langle\xi^3\rangle_{\phi_{T_\perp}}$	0.0665	$\langle\xi^2\rangle_{\phi_{\rm even}}$	0.0226
$\langle\xi^5\rangle_{\phi_{\rm odd}}$	0.00685	$\langle\xi^5\rangle_{\phi_T}$	0.00735	$\langle\xi^5\rangle_{\phi_{T_\perp}}$	0.00684	$\langle\xi^4\rangle_{\phi_{\rm even}}$	0.00142
$\langle\xi^7\rangle_{\phi_{\rm odd}}$	0.000922	$\langle\xi^7\rangle_{\phi_T}$	0.00102	$\langle\xi^7\rangle_{\phi_{T_\perp}}$	0.000919	$\langle\xi^6\rangle_{\phi_{\rm even}}$	0.000139

Table 3. The ξ-moments for the p-wave bottomonium states in this work.

of quark-antiquark pair inside the p-wave charmonium state to the ξ-moments as:

$$\langle v^n\rangle_p = \frac{n+3}{3}\langle\xi^{n+1}\rangle + O(v^{n+2}) \tag{4.6}$$

If the ξ-moments $\langle\xi^n\rangle_{\phi_{\rm odd}}$ are considered, we obtain:

$$\begin{aligned}\langle v^2\rangle_p &= 0.317,\\ \langle v^4\rangle_p &= 0.118,\\ \langle v^6\rangle_p &= 0.0492,\end{aligned}$$

for the charmonium sector and

$$\begin{aligned}\langle v^2\rangle_p &= 0.111,\\ \langle v^4\rangle_p &= 0.0160,\\ \langle v^6\rangle_p &= 0.00277,\end{aligned}$$

for the bottomonium sector. These results are consistent with the values $\langle v^2\rangle_{cc} \approx 0.3$ and $\langle v^2\rangle_{bb} \approx 0.1$ used in NRQCD.

5 Conclusions

This study discussed the leading twist LCDAs of the p-wave heavy quarkonium states within the light-front approach. The twist-2 LCDAs have been disentangled from the

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higher twists by appropriately coping with the nonlocal operators $\bar{q}(-z)\Gamma q(z)$. For the $\Gamma = \sigma_{\mu\nu}(\gamma_5)$ case, we proved that our method is equivalent to that of ref. [31]. Next, these LCDAs have been shown in terms of the light-front variables $(u, \kappa_\perp)$ and the relevant decay constants. We found that the decay constants and LCDAs had the following relations: $\sqrt{3}f_S = f_{^1A_1} = \sqrt{2}f^\perp_{^3A_1}(\equiv f_{\rm odd})$, $f_{^3A_1}/\sqrt{2} = f^\perp_{^1A_1}(\equiv f_{\rm even})$, and $\phi_S = \phi_{^1A_1\parallel} = \phi_{^3A_1\perp}(\equiv \phi_{\rm odd})$, $\phi_{^3A_1\parallel} = \phi_{^1A_1\perp}(\equiv \phi_{\rm even})$. If one takes the non-relativistic limit and the wave function as a function of $|\vec{\kappa}|$, then the above relations among the decay constants could be further simplified as $f_{\rm odd} \simeq f_T \simeq f^\perp_T \simeq f_{\rm even}$, and in addition, the ξ-moments of $\phi_{\rm odd}$ and $\phi_{\rm even}$ have the relation: $\langle\xi^n\rangle_{\phi_{\rm even}} = \langle\xi^n\rangle_{\phi_{\rm odd}}/(n+1)$.

The $\kappa_\perp$ integrations for the equations of LCDAs and ξ-moments could be analytically performed when the Gaussian-type wave function is considered. The parameters m and β, which appear in the wave function, were determined by taking the mass of the spin-weighted average of the triplet state $M(^3P_J)$ and the variational principle for its Hamiltonian into account. We found the parameters m and β insensitively depended on the linear potential constant b and the strong coupling constant α_s. The curves and the corresponding decay constants of the LCDAs $\phi_{\rm odd}$, $\phi_{T\parallel}$, $\phi_{T\perp}$, and $\phi_{\rm even}$ were plotted and calculated for the charmonium and bottomonium states. These results are consistent with the relations which are mentioned in last paragraph. However, the value of $f_{\rm odd}$ is about a factor of two smaller than that in [6] which was studied within QCD sum rules. In addition, the first three ξ-moments were calculated, and were consistent with those of other theoretical approaches. The relative velocity of quark-antiquark pair $\langle v^2\rangle$ of charmonium and bottomonium states were also estimated and were consistent with those used in NRQCD.

Acknowledgments

The author would like to thank Kwei-Chou Yang and A. V. Luchinsky for their helpful discussions. This work is supported in part by the National Science Council of R.O.C. under Grant No NSC-96-2112-M-017-002-MY3.

A Derivations of eqs. (2.20)–(2.22)

Firstly, eqs. (2.8), (2.9), and (2.11) are rewritten as

$$\langle 0|\bar{q}(z)\gamma_\mu q(-z)|S(P)\rangle = f_S \int_0^1 du\; e^{i\xi pz}\left\{P_\mu\phi_S(u) + z_\mu\frac{M_S^2}{2pz}\left[g_S(u) - \phi_S(u)\right]\right\}, \quad (A.1)$$

$$\langle 0|\bar{q}(z)\gamma_\mu\gamma_5 q(-z)|A(P,\epsilon_{\lambda=0})\rangle = if_A M_A \int_0^1 du\; e^{i\xi pz}\Big\{\epsilon_\mu\phi_{A\parallel}(u) + \epsilon_{\perp\mu}\left[g_{A\perp}(u) - \phi_{A\parallel}(u)\right] - z_\mu\frac{\epsilon z}{2(pz)^2}M_A^2\left[g_{A3}(u) - \phi_{A\parallel}(u)\right]\Big\}, \quad (A.2)$$

$$\langle 0|\bar{q}(z)\gamma_\mu q(-z)|T(P,\epsilon_{\lambda=0})\rangle = f_T M_T^2 \int_0^1 du\; e^{i\xi pz}\Big\{\frac{\epsilon_{\mu\bullet}}{pz}\phi_{T\parallel}(u) + \frac{\epsilon_{\perp\mu\bullet}}{pz}\left[g_{T\perp}(u) - \phi_{T\parallel}(u)\right] - z_\mu\frac{\epsilon_{\bullet\bullet}}{2(pz)^3}M_T^2\left[g_{T3}(u) - \phi_{T\parallel}(u)\right]\Big\}, \quad (A.3)$$

respectively. Next, we sandwich both sides of eq. (2.19) between the vacuum and, for example, the scalar meson state

$$\begin{aligned}\langle 0|[\bar{q}(-z)\gamma_\mu q(z)]_2|S(P)\rangle &= \int_0^1 dt\frac{\partial}{\partial z^\mu}\langle 0|\bar{q}(-tz)\not{z}q(tz)|S(P)\rangle \\ &= f_S\int_0^1 dt\frac{\partial}{\partial z^\mu}Pz\int_0^1 du e^{i\xi pz}\phi_S(u) \\ &= f_S\int_0^1 du\phi_S(u)\left\{P_\mu\int_0^1 dt e^{i\xi tpz}+p_\mu(i\xi Pz)\int_0^1 dtte^{i\xi tpz}\right\}.\end{aligned} \tag{A.4}$$

The second term of the last line can be further calculated as:

$$p_\mu(i\xi Pz)\int_0^1 dtte^{i\xi tpz}=p_\mu\frac{Pz}{pz}\int_0^1 dtt\frac{\partial}{\partial t}e^{i\xi tpz}=p_\mu\left[e^{i\xi pz}-\int_0^1 dte^{i\xi tpz}\right]. \tag{A.5}$$

We can substitute eq. (A.4) for eq. (A.5) and obtain eq. (2.20). In addition, the same process can be used to obtain eqs. (2.21) and (2.22). In fact, the above process has been used for the vector meson state in ref. [32].

B Derivations of eqs. (2.31) and (2.32)

Eqs. (2.10) and (2.12) can be rewritten as:

$$\begin{aligned}\langle 0|\bar{q}(z)\sigma_{\mu\nu}\gamma_5 q(-z)|A(P,\epsilon_{\lambda=\pm1})\rangle &= f_A^\perp\int_0^1 du\ e^{i\xi pz}\Big\{(\epsilon_\mu P_\nu-\epsilon_\nu P_\mu)\phi_{A\perp}(u) \\ &\quad+(p_\mu z_\nu-p_\nu z_\mu)\frac{M_A^2\epsilon z}{(pz)^2}[h_{A\parallel}(u)-\phi_{A\perp}(u)] \\ &\quad+(\epsilon_{\perp\mu}z_\nu-\epsilon_{\perp\nu}z_\mu)\frac{M_A^2}{2pz}[h_{A3}(u)-\phi_{A\perp}(u)]\Big\},\end{aligned} \tag{B.1}$$

$$\begin{aligned}\langle 0|\bar{q}(z)\sigma_{\mu\nu}q(-z)|T(P,\epsilon_{\lambda=\pm1})\rangle &= if_T^\perp M_T\int_0^1 du\ e^{i\xi pz}\Big\{\frac{(\epsilon_{\mu\bullet}P_\nu-\epsilon_{\bullet\nu}P_\mu)}{pz}\phi_{T\perp}(u) \\ &\quad+(p_\mu z_\nu-p_\nu z_\mu)\frac{M_T^2\epsilon_{\bullet\bullet}}{(pz)^3}[h_{T\parallel}(u)-\phi_{T\perp}(u)] \\ &\quad+(\epsilon_{\perp\mu\bullet}z_\nu-\epsilon_{\perp\bullet\nu}z_\mu)\frac{M_T^2}{2(pz)^2}[h_{T3}(u)-\phi_{T\perp}(u)]\Big\},\end{aligned} \tag{B.2}$$

respectively. Then, we sandwich both sides of eq. (2.30) between the vacuum and, for example, the axial-vector meson state,

$$\begin{aligned}&\langle 0|[\bar{q}(-z)\gamma_\mu\gamma_5 q(z)]_2|A(P,\epsilon_{\lambda=\pm1})\rangle \\ &= \int_0^1 dt\left[\frac{\partial}{\partial z^\mu}\langle 0|\bar{q}(-t^2z)\sigma_{\bullet\nu}\gamma_5 q(t^2z)|A(P,\epsilon)\rangle+z^\alpha\frac{\partial}{\partial z^\nu}\langle 0|\bar{q}(-t^2z)\sigma_{\mu\alpha}\gamma_5 q(t^2z)|A(P,\epsilon)\rangle\right] \\ &= f_A^\perp\int_0^1 du\Big\{\phi_{A\perp}(u)\left[(\epsilon_\mu P_\nu-\epsilon_\nu P_\mu)\int_0^1 dte^{i\xi t^2pz}+2pz\mathcal{S}_{\mu\nu}(i\xi)\int_0^1 dtt^2e^{i\xi t^2pz}\right] \\ &\qquad+\big(h_{A\parallel}(u)-\phi_{A\perp}(u)\big)\left[\mathcal{U}_{\mu\nu}\int_0^1 dte^{i\xi t^2pz}+2pz\mathcal{T}_{\mu\nu}(i\xi)\int_0^1 dtt^2e^{i\xi t^2pz}\right]\Big\}.\end{aligned} \tag{B.3}$$

We can further calculate the integral as:

$$i\xi\int_0^1 dtt^2e^{i\xi t^2pz}=\frac{1}{2pz}\int_0^1 dtt\frac{\partial}{\partial t}e^{i\xi t^2pz}=\frac{1}{2pz}\left[e^{i\xi pz}-\int_0^1 dte^{i\xi t^2pz}\right], \tag{B.4}$$

and then substitute eq. (B.3) for eq. (B.4) to obtain eq. (2.31). The same process can be used to obtain eq. (2.32).

C Derivations of eqs. (3.35) and (3.36)

From eqs. (3.32), (3.34) and the normalization eq. (2.13), we have

$$f_{\rm odd}\simeq\sqrt{6}\int_{-1}^1 d\xi\xi\int\frac{d^2\kappa_\perp}{2(2\pi)^3}\frac{\xi}{\sqrt{1-\xi^2}}\frac{M_0}{2}\varphi_p(\xi,\kappa_\perp), \tag{C.1}$$

$$f_{\rm even}=\sqrt{6}\int_{-1}^1 d\xi\int\frac{d^2\kappa_\perp}{2(2\pi)^3}\frac{1}{\sqrt{1-\xi^2}}\frac{\kappa_\perp^2}{M_0}\varphi_p(\xi,\kappa_\perp). \tag{C.2}$$

Taking eqs. (C.1) and (C.2) integration by parts with respect to $\kappa_\perp^2$ and ξ, respectively, and using normalization conditions: $\varphi_p(\xi,\kappa_\perp)$ must go to zero when ξ ($\kappa_\perp$) go to ±1 (infinity), and thus, we can obtain:

$$f_{\rm odd}\simeq-\sqrt{6}\int_{-1}^1 d\xi\xi\int\frac{d^2\kappa_\perp}{2(2\pi)^3}\kappa_\perp^2\frac{\xi}{\sqrt{1-\xi^2}}\frac{d}{d\kappa_\perp^2}\left[\frac{M_0}{2}\varphi_p(\xi,\kappa_\perp)\right], \tag{C.3}$$

$$f_{\rm even}=-\sqrt{6}\int_{-1}^1 d\xi\xi\int\frac{d^2\kappa_\perp}{2(2\pi)^3}\kappa_\perp^2\frac{d}{d\xi}\left[\frac{1}{\sqrt{1-\xi^2}}\frac{1}{M_0}\varphi_p(\xi,\kappa_\perp)\right]. \tag{C.4}$$

From eq. (3.13), the differentiations in eqs. (C.3) and (C.4) can be expanded as:

$$\frac{\xi}{\sqrt{1-\xi^2}}\frac{d}{d\kappa_\perp^2}\left[\frac{M_0}{2}\varphi_p(\xi,\kappa_\perp)\right]=\frac{3\xi}{2\sqrt{M_0}(1-\xi^2)^2}F(\vec{\kappa})+\frac{\xi M_0^{3/2}}{2(1-\xi^2)}\frac{d}{d\kappa_\perp^2}F(\vec{\kappa}), \tag{C.5}$$

$$\frac{d}{d\xi}\left[\frac{1}{\sqrt{1-\xi^2}}\frac{1}{M_0}\varphi_p(\xi,\kappa_\perp)\right]=\frac{3\xi}{2\sqrt{M_0}(1-\xi^2)^2}F(\vec{\kappa})+\frac{1}{\sqrt{M_0}(1-\xi^2)}\frac{d}{d\xi}F(\vec{\kappa}), \tag{C.6}$$

respectively. If the function $F=F(|\vec{\kappa}|)$, one can expand the arbitrary function $F(|\vec{\kappa}|)$ as a polynomial of $|\vec{\kappa}|$:

$$F(|\vec{\kappa}|)=\sum_s c_s|\vec{\kappa}|^s. \tag{C.7}$$

Using the relation $|\vec{\kappa}|=\sqrt{M_0^2/4-m^2}$, one can calculate the differentiations in eqs. (C.5) and (C.6) as:

$$\frac{\xi M_0^{3/2}}{2(1-\xi^2)}\frac{d}{d\kappa_\perp^2}F(|\vec{\kappa}|)=\frac{1}{\sqrt{M_0}(1-\xi^2)}\frac{d}{d\xi}F(|\vec{\kappa}|)=\sum_s c_s\frac{\xi s M_0^{3/2}}{4(1-\xi^2)^2}\left(\frac{M_0^2}{4}-m^2\right)^{s/2-1} \tag{C.8}$$

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Then, eq. (3.35) is obtained. Regarding the ξ-moments of $\phi_{\rm odd}$ and $\phi_{\rm even}$, we have:

$$\langle \xi^{n+1} \rangle_{\phi_{\rm odd}} \simeq \frac{\sqrt{6}}{f_{\rm odd}} \int_{-1}^{1} d\xi \xi^{n+1} \int \frac{d^2\kappa_\perp}{2(2\pi)^3} \frac{\xi}{\sqrt{1-\xi^2}} \frac{M_0}{2} \varphi_p(\xi, \kappa_\perp), \tag{C.9}$$

$$\langle \xi^{n} \rangle_{\phi_{\rm even}} = \frac{\sqrt{6}}{f_{\rm even}} \int_{-1}^{1} d\xi \xi^{n} \int \frac{d^2\kappa_\perp}{2(2\pi)^3} \frac{1}{\sqrt{1-\xi^2}} \frac{\kappa_\perp^2}{M_0} \varphi_p(\xi, \kappa_\perp). \tag{C.10}$$

Using the above processes, eq. (3.36) can be obtained.

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